

Proposal Babyseminar on Reductive Groups

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We will assume k to be an algebraic closed field of any characteristic. The main goal of the seminar will be to study first the structure of reductive groups and then a bit of their representation theory. In the first part we recall some definitions and properties of algebraic groups, talk about Borel subgroups and flag variety, weyl group,... Then introduce reductive algebraic groups and prove the theorem on the classification of reductive groups in terms of root data. For the second part, I would like to see the classification of the semisimple representations (split) reductive groups, in terms of dominant weights of a maximal torus, for example in Part II, Chapter 2 of [Jan07]. In particular, when k has characteristic zero, this includes all of them. Then, time permitting, I would like to see something about the characteristic p case. Throughout the seminar, I would like to look at basic examples while studying the general theory.

For the first part some possible references are [Spr94], [Hum12], and for the second part a possible reference is Part II, Chapters 1-3 of [Jan07], or Chapter 22 of [Mil17] and use [FH99] for examples.

References

- [FH99] William Fulton and Joe Harris. *Representation theory*. Graduate Texts in Mathematics. Springer, 1999.
- [Hum12] James E Humphreys. *Linear Algebraic Groups*. Graduate Texts in Mathematics. Springer, 2012. URL: <https://link.springer.com/book/10.1007/978-1-4684-9443-3#accessibility-information>.
- [Mil17] J. S. Milne. *Algebraic Groups: The Theory of Group Schemes of Finite Type over a Field*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 2017.
- [Spr94] T. A. Springer. “Linear Algebraic Groups”. In: *Algebraic Geometry IV: Linear Algebraic Groups Invariant Theory*. Ed. by A. N. Parshin and I. R. Shafarevich. Berlin, Heidelberg: Springer Berlin Heidelberg, 1994, pp. 1–121. ISBN: 978-3-662-03073-8. DOI: 10.1007/978-3-662-03073-8_1. URL: https://doi.org/10.1007/978-3-662-03073-8_1.