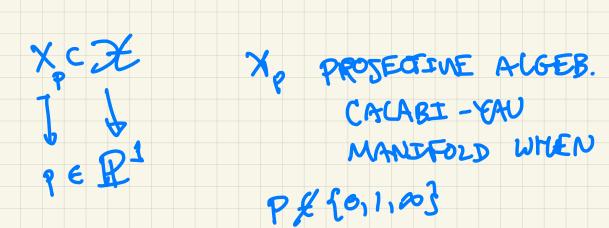
ANS: SEMINAL WORK OF GREENE-PLESSER & CANDELAS - DELAOSSA - GREEN-PARKES

ONE ANOTHER?

Q: WHAT DOTHESE HAVE TO DO WITTH

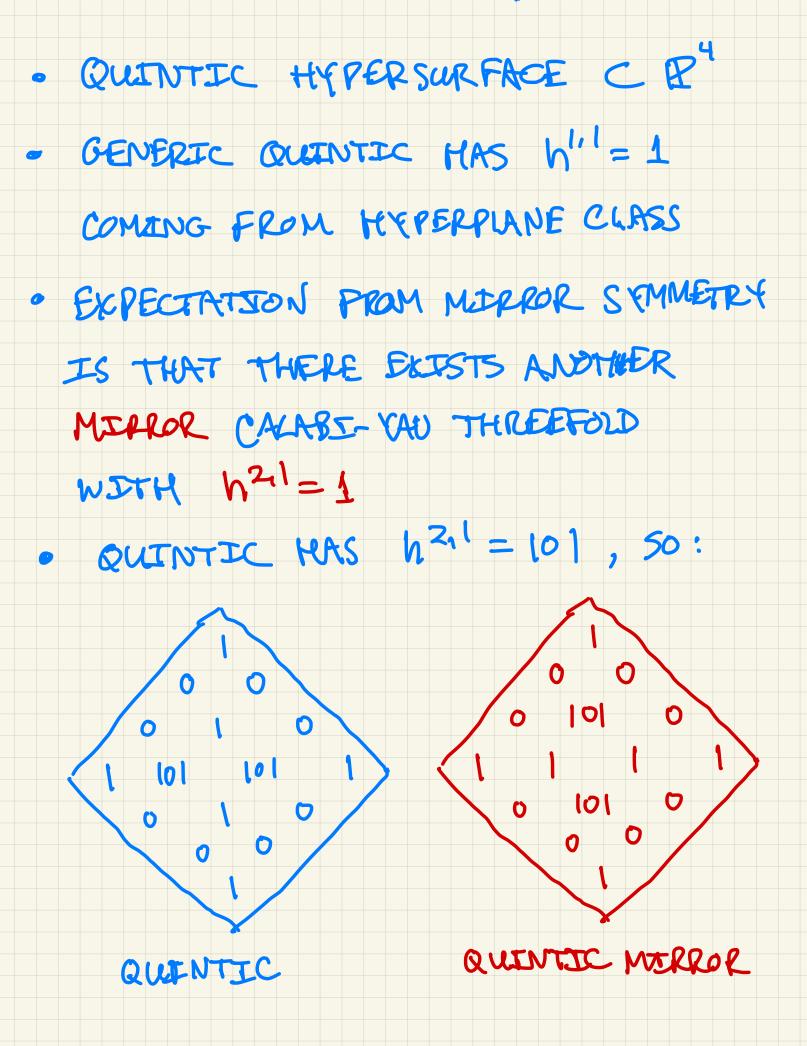


· MIRROR SYMMETRY

PUNCTURED SPHERE

FAMILIES OVER THE THRICE

MIRROR SYMMETRY AND CALABI-YAU



<u>GREENE - PLESSER</u>: STARTING FROM <u>GEPNER</u> MODELS IN <u>CFT</u> PROPOSED <u>A CONSTRUCTION</u> OF THE MIPROR $\frac{4}{2}$ $x_{i}^{5} = 0$ / QUOTLENT BY ACTION FERMAT

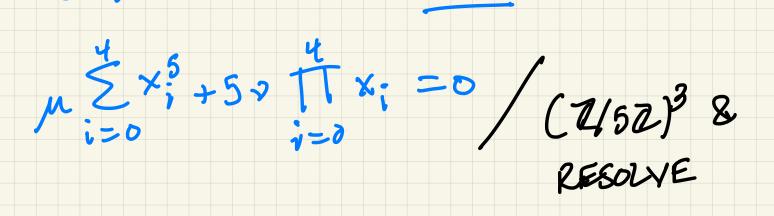
QUINTER RESOLVE SENGS

THAT GEVES ONE CX 3-FOLD WITH

THE CORRECT HODGE NUMBERS.

THE OTHERS ARAGE BY DOING THE SAME

CONSTRUCTION IN A FAMILLE



· CONSTRUCTED IN THIS WAY	A FAMOLY
OF SMOOTH, PROJECTIVE CACK	FBI-YAU
3-FOLDS OVER R'-50,1,20	3
CANDELLS - DELLA OSSA - COZEEN - PARKE	
· STUDIED INTEGRALS OF W ^(3,0)	orer
INTEGRAL 3-CROCES ON THE	x 3-FOLRS
SIN THIS FAMELY	
• PERJODS SATISFY GENERALSZE	
HYPEROEOMETRIC ODE ANNI	MILATING
$_{4}F_{3}(\pm, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}; 1, 1, 1, \frac{1}{2})$	
FOR ZGRI BASE OF FAMIL	*
· THREE REGULAR SINGULAR P	OTUTS
	\otimes
MAX UNIPOTENT CONSTOLD O MONODROMY MONODROMY	MONODROMY

- HARD PROBLEM () EASY PROBLEM
- · MIREOR SYMMETRY OFTEN TAKES

O I OO MAX UNIPOTENT CONIFOLD QUASI-UNIPOTENT MONODROMY MONODROMY MONODROMY

- FAMILIES OF CALABE- (AU 3-FOLDS WITH $h^{2,1} = 1$ OVER $\mathbb{P}^1 \setminus \{0, 1, \infty\}$
- SIMPLEST MIRRORS SHOUD BE
- SHOULD APPLY TO ANYE CY 3-FOLD
- CORRESPOND TO INTEGRAL MONODROMILES ABOUT O, 1, 00 IN MIRROR QUINTIC FAMILY
- CORRESPOND TO INTEGRAL MONODROMIES
- · AUTOMOGRYHIJSMS OF D' (OUENTIC)

KONTSENSCH'S HMS:

D-MORGAN:

O

- WHAT ABOUT MIRRORING THE HARD PROBLEM OF FINDENG ALL CALABI-VAD 3-FOLDS WITTH $h_{1}^{1,1} = 1$?
- MIRROR PROBLEM : FIND ALL
 - CALABI-VAU 3-FOLDS WITH N21 = 1
- · SIMPLEST NONTRINGAL CASE WOULD
 - HEAVE BASE EP1-291,003
- · TO BE COMPATIBLE WITH KONTSEVECH
 - PREDICTION SHOULD LOOK FOR
 - Z-VHS OVER B' 50,1,003 WITH
 - MAX UNIPOTENT CONTROLD QUAST-UNIPOTENT MONODROMY MONODROMY MONODROMY

 \bigotimes

1

• IS THIS EVEN A FINITE PROBLEM?

• DELIGNE: FINITENESS OF Z-VHS & EQUINALENCE WITH CLASSIFICATION OF INTEGRAL SYMPLECTIC MONODRAMY REPRESENTATIONS $\rho: \pi_{1}(\mathbb{P}^{\prime} \setminus \{0, 1, \infty\}, \mathbb{Z}) \longrightarrow S_{p}(\mathcal{A}, \mathbb{Z})$ · D-MORGAN : CLASSIFICATION OF WEIGHT 3 HODGE TYPE (1,1,1,1) Z-VHS OVER [P-50,1,00] WITH 0 \bigotimes 1 CONTROLD QUAST-UNTROTENT MAX UNIPOTENT MONODROMY MONODROMY MONODROMY ID 112 POSSIBLE Z-VHS UP TO CONJUGATION EQUEVALENCE 23 OF THESE ARE "MIRROR COMPATIBLE" IN A STRANG SENSE 14 R-VHS

m	a	a_1, a_2, a_3, a_4	$\# L_{\mathbb{Z}}$	$\# L_{\rm MC}$	t	Geometric examples
1	4	$\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$	1	1	1	Ι
1	3	$\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}$	1	1	1	$\mathbb{WP}^4_{1,1,1,2,5}[10]$
2	4	$\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$	2	2	1	$\mathbb{WP}^4_{1,1,1,1,4}[8]$
					2	II
5	5	$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$	2	2	1	$\mathbb{P}^4[5]$
					5	А
1	2	$\frac{1}{6}, \frac{1}{6}, \frac{5}{6}, \frac{5}{6}$	1	1	1	$\mathbb{WP}^5_{1,1,2,2,3,3}[6,6]$
2	3	$\frac{1}{6}, \frac{1}{4}, \frac{3}{4}, \frac{5}{6}$	1	1	1	$\mathbb{WP}^5_{1,1,1,2,2,3}[4,6]$ *
3	4	$\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{5}{6}$	1	1	1	$\mathbb{WP}^4_{1,1,1,1,2}[6]$
4	5	$\frac{1}{6}, \frac{1}{2}, \frac{1}{2}, \frac{5}{6}$	11	1	1	$\mathbb{WP}^5_{1,1,1,1,1,3}[2,6]$
4	4	$rac{1}{4},rac{1}{4},rac{3}{4},rac{3}{4}$	8	3	1	$\mathbb{WP}^5_{1,1,1,1,2,2}[4,4]$
					2	В
					4	С
6	5	$\frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}$	1	1	1	$\mathbb{WP}^5_{1,1,1,1,1,2}[3,4]$ *
8	6	$\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}$	14	2	1	$\mathbb{P}^5[2,4]$
					2	D
9	6	$\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}$	8	2	1	$\mathbb{P}^5[3,3]$
					3	Е
12	7	$\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{2}{3}$	11	1	1	$\mathbb{P}^6[2,2,3]$
16	8	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	50	4	1	$\mathbb{P}^{7}[2, 2, 2, 2]$
					2	F
					4	G
					8	Н

TABLE 1. Table of "Mirror-Consistent" $\rho \colon \pi_1(\mathbb{P}^1 \setminus \{0, 1, \infty\}) \to Sp(4)$

· QUESTION REMAINS OF GEOMETRIC REALIZATION OF EACH Z-VHS BY AN ACTUAL FAMILY OF PROJECTIVE ACCEBRAIC CALABI-YAU 3-FOLDS WITH h21 = 1 FOR 13 OF THE 14 R-UNS, THISS • WAS EASY, AS EXISTING TORIC HYPERSURFACE OR CI CONSTRUCTIONS 2 BATYREN-BORISON MIRROR SOMMETRY GAVE THE FAMILY · "14th CASE" WAS MUCH MODE SUBTLE 4F3 (12, 52, 72, 112; 1, 1, 1 (2)

· NOT DIPFICULT TO SEE THAT THERE IS A 3-PARAMETER FAMILY OF h21 = 3 CALABE-VAU 3-FOLDS AS THE MIRRAR OF [2,12] CICY IN $WP^{2}(1,1,2,8,12)$ WHOSE GKZ-HYPERBEDMETRIC PERTOD FUNCTIONS RESTRICT TO THE DESIRED 4F3 (12, 5, 7, 12; 1, 1, 1 2) ALONG A 1-PARAMETER SUBFAMILY · BUT : EVERY WHERE ALONG THIS 1-PARAMETER SUBFAMILY THE MTREOR CECK BECOME SINGULAR · TO FHIS TOXY : NO KNOWN SMOOTH FAMELY OF PROJECTIVE ALGEBRATE CY 3FOLDS THAT REMADE Z-VHS!

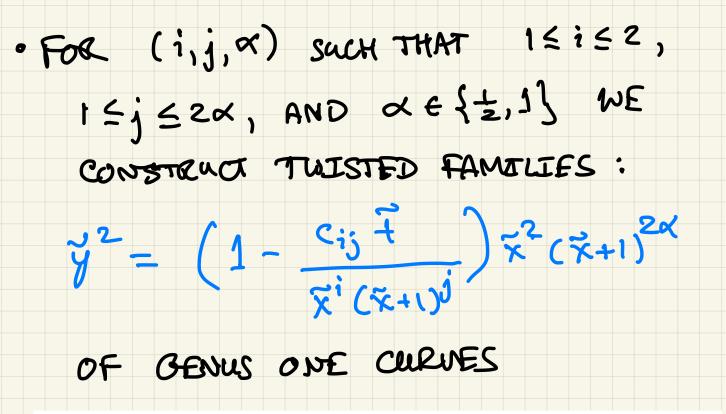
CLIVOHER-D.-LEWIS-NOVOSELTSEV-THOMPSON:

- EVEN AFTER CREPANT PARTIAL DESINGULARIZATION, CENERIC CL3 IN FAMILY HAS A PAIR OF R-FACTORIAL NODAL SINGULARITIES
- · NOT EVEN BIRATIONAL TO SMOOTH CY3 REALIZING THE Z-VHS DERECTLY
- SHOW CAN GEOMETRICALLY REALIZE Z-VHS, AS 3rd WEIGHT ORADED PIECE OF MIKED MODGE STRUCTURE ON H³ ADMITS A PURE Z-VHS OF WT. 3 & HODGE TYPE (1,1,1,1) THAT
 - REALIZES THE 14th CASE Z-VHS.
- CONSTRUCTED N PROVIDES A
 COUNTER EXAMPLE TO A CONSECTURE
 OF NORRISON ON MIRROR OF DEGEN >

Q: WHAT GEOMETRICALLY DO ALL OF THE "MERROR" CX 3-FOLDS HAVE IN COMMON NOT A: THEY ARE MERRORS OF OUPRTAZ BETTER ADSWER COMES FROM OBSERVING INTERNAL STRUCTURE OF THESE CR3FOLDS · D- HARDER-NOVOSELTSEN-THOMPSON THEY ARE FIBERED BY HIGH PICARD RANK (180219) K3 SURFACES · D.-MALMENDJER THEY ARE FIBERED (POSSIBLE IN MULTIPLE WARS) BY Mn - POLARIZED K3 SURFACES = MIRRORS TO (2n)-POLARIZED K3 SURFACES

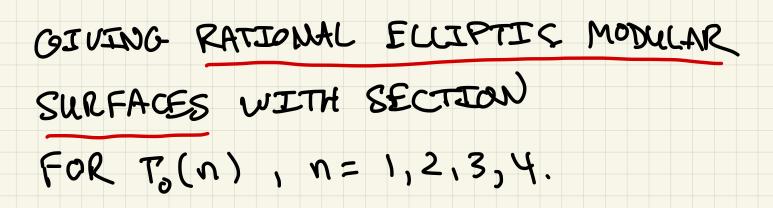
- D-HARDER-NOVOSELTSEN-THOMPSON THEY ARE FIBERED BY HIGH PICARD RANK (180217) K3 SURFACES
- METHOD : SEARCH FOR TORICALLY INDUCED FIBRATIONS ON C'S 3-FOLD HYPER-SURFACES OR CI'S & RECOGNIZE FIBERS AS K3 SURFACE HYPER-SURFACES OR CI'S IN OTHER TORIC NARIETIES
 - MOTIVATED & PROGRAM (SUCCESSFUL!)
 TO COMPLETELY CLASSEFY & CONFLUCT
 ALL CY 3-FOLDS THAT ARE FIBERED
 BX Mn-POLARIZED F3 SURFACES

· D.-MALMENDJER THEY ARE FIBERED (POSSIBLY IN MULTIPLE WARS) BY Mn-POLARIZED K3 SURFACES = MIRRORS TO (2N)-POLARIZED K3 SURFACES · METHOD : ITERATIVELY CONSTRUCT FAMILIES OF CY N-FOLDS FROM FAMELSES OF CY (1-1)-FOLDS · STHRTING POINT: CY O-FOLDS = PAER OF POINTS = SOLUTION TO $y^2 = 1 - t$ $\int \pm y \int$ WITH PERIOD ,Fo(之(七)



(i, j, α)	$\mid \mu$	singular fibers	notation
(1, 1, 1)	$\frac{1}{2}$	$I_1^*(\tilde{t} = \infty), \ I_4(\tilde{t} = 0), \ I_1(\tilde{t} = 1)$	X ₁₄₁
(2, 1, 1)	$\frac{1}{3}$	$IV^*(\tilde{t} = \infty), \ I_3(\tilde{t} = 0), \ I_1(\tilde{t} = 1)$	X_{431}
$(1,1,\frac{1}{2})$	$\frac{1}{4}$	$III^{*}(\tilde{t} = \infty), I_{2}(\tilde{t} = 0), I_{1}(\tilde{t} = 1)$	X_{321}
$\left(2,1,\frac{1}{2}\right)$	$\frac{1}{6}$	$II^*(\tilde{t} = \infty), \ I_1(\tilde{t} = 0), \ I_1(\tilde{t} = 1)$	X_{211}

 Table 4: Families of elliptic curves



· AT THE NEXT STEP WE TWIST UP AGAIN

derive	ed from	ρ	Configuration of singular fibers			$MW(\pi,\sigma)$	$\operatorname{disc} Q$	Λ
Srfc	μ		$u = \infty$	$u^2 + u + t/4$	u = 0, -1			
X_{141}	$\frac{1}{2}$	19	$I_8(A_7)$	$2 I_1$	$2 I_1^* (2 D_5)$	[4]	2^{3}	M_4
X_{431}	$\frac{1}{3}$	19	$I_6 (A_5)$	$2 I_1$	$2 IV^* (2 E_6)$	[3]	$2 \cdot 3$	M_3
X_{321}	$\frac{1}{4}$	19	$I_4(A_3)$	$2 I_1$	$2III^{*}(2E_{7})$	[2]	2^{2}	M_2
X_{211}	$\frac{1}{6}$	19	$I_2(A_1)$	$2 I_1$	$2 II^* (2 E_8)$	[1]	2	M_1

Table 7: K3 surfaces from extremal rational surfaces

FIELDING WEDERSTRASS ELLIPTIC FIBERED LATTICE - POLARIZED 43 SURFACES WITH GENEREC PICARD LATTICE OF TYPE $M_{n} := H \oplus E_8 \oplus E_8 \oplus (-2n), n = 1,2,3,4$ · AGAIN THE FAMILY OF K3 SURFACES IS FIBERED OVER THE THRICE PUNCTURED SPHERE

· ONCE MOBE AND WE GET ALL 60 SEMPLECTICALLE RIGID Z-VHS CLASSIFIED BY BOONER-PETTER, INCLUDENCE THE 14 R-VHS OF HYPEROEDMETRIC TYPE · EXPLICIT WEDERSTRASS EULIPTIC Mn- POLARIZED K3 SURFACE-FIBERED CALABI-XAU 3-FOLDS · AGAIN ARE FAMILLES OVER THE THRICE-PUNCTURED SPILERE! • NO NEED TO STOP THERE ... IN FACT OET INFFNITE ON TOWERS OVER THE THRICE PUNCTURED SPHERE

MOST FAMORES SOURCE OF CY MIRROR

MANJEOLOS IN ALL DEMINS:

DEOREE (n+1) HYPERSECES IN Bn : Xn-1

MIRROR TO Xn-1 IS FIBERED

BY MERROR TO Xn-2!

COMMENT: THE ACTUAL PROOF

OF CONSTR. OF PERSONS OVER

2-CYCLES IN CLOBP HAS

A HIDDEN REALEZ. OF "QUARTIC

MIRROR FIRRATION ON QUENTIC MJEROR"